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A BRIEF HISTORY OF FEEDBACK CONTROL

Contents

Outline

In this chapter we introduce modern control theory by two approaches. First, a short history of automatic control theory is provided. Then, we describe the philosophies of classical and modern control theory.

Feedback control is the basic mechanism by which systems, whether mechanical, electrical, or biological, maintain their equilibrium or homeostasis. In the higher life forms, the conditions under which life can continue are quite narrow. A change in body temperature of half a degree is generally a sign of illness. The homeostasis of the body is maintained through the use of feedback control [Wiener 1948]. A primary contribution of C.R. Darwin during the last century was the theory that feedback over long time periods is responsible for the evolution of species. In 1931 V. Volterra explained the balance between two populations of fish in a closed pond using the theory of feedback.

Feedback control may be defined as the use of difference signals, determined by comparing the actual values of system variables to their desired values, as a means of controlling a system. An everyday example of a feedback control system is an automobile speed control, which uses the difference between the actual and the desired speed to vary the fuel flow rate. Since the system output is used to regulate its input, such a device is said to be a *closed-loop control system*.

In this book we shall show how to use *modern control theory* to design feedback control systems. Thus, we are concerned not with natural control systems, such as those that occur in living organisms or in society, but with man-made control systems such as those used to control aircraft, automobiles, satellites, robots, and industrial processes.

Realizing that the best way to understand an area is to examine its evolution and the reasons for its existence, we shall first provide a short history of automatic control theory. Then, we give a brief discussion of the philosophies of classical and modern control theory.

The references for Chapter 1 are at the end of this chapter. The references for the remainder of the book appear at the end of the book.

1.1 A BRIEF HISTORY OF AUTOMATIC CONTROL

There have been many developments in automatic control theory during recent years. It is difficult to provide an impartial analysis of an area while it is still developing; however, looking back on the progress of feedback control theory it is by now possible to distinguish some main trends and point out some key advances.

Feedback control is an engineering discipline. As such, its progress is closely tied to the practical problems that needed to be solved during any phase of human history. The key developments in the history of mankind that affected the progress of feedback control were:

1. The preoccupation of the Greeks and Arabs with keeping accurate track of time. This represents a period from about 300 BC to about 1200 AD.

2. The Industrial Revolution in Europe. The Industrial Revolution is generally agreed to have started in the third quarter of the eighteenth century; however, its roots can be traced back into the 1600's.

3. The beginning of mass communication and the First and Second World Wars. This represents a period from about 1910 to 1945.

4. The beginning of the space/computer age in 1957.

One may consider these as phases in the development of man, where he first became concerned with understanding his place in space and time, then with taming his environment and making his existence more comfortable, then with establishing his place in a global community, and finally with his place in the cosmos.

At a point between the Industrial Revolution and the World Wars, there was an extremely important development. Namely, control theory began to acquire its written language- the language of mathematics. J.C. Maxwell provided the first rigorous mathematical analysis of a feedback control system in 1868. Thus, relative to this written language, we could call the period before about 1868 the *prehistory* of automatic control.

Following Friedland [1986], we may call the period from 1868 to the early 1900's the *primitive period* of automatic control. It is standard to call the period from then until 1960 the *classical period*, and the period from 1960 through present times the *modern period*.

Let us now progress quickly through the history of automatic controls. A reference for the period -300 through the Industrial Revolution is provided by [Mayr 1970], which we shall draw on and at times quote. See also [Fuller 1976]. Other important references used in preparing this section included [M. Bokharaie 1973] and personal discussions with J.D. Aplevich of the University of Waterloo, K.M. Przyluski of the Polish Academy of Sciences, and W. Askew, a former Fellow at LTV Missiles and Space Corporation and vice-president of E-Systems.

Water Clocks of the Greeks and Arabs

The primary motivation for feedback control in times of antiquity was the need for the accurate determination of time. Thus, in about -270 the Greek Ktesibios invented a *float regulator* for a water clock. The function of this regulator was to keep the water level in a tank at a constant depth. This constant depth yielded a constant flow of water through a tube at the bottom of the tank which filled a second tank at a constant rate. The level of water in the second tank thus depended on time elapsed.

The regulator of Ktesibios used a float to control the inflow of water through a valve; as the level of water fell the valve opened and replenished the reservoir. This float regulator performed the same function as the ball and cock in a modern flush toilet.

A float regulator was used by Philon of Byzantium in -250 to keep a constant level of oil in a lamp.

During the first century AD Heron of Alexandria developed float regulators for water clocks. The Greeks used the float regulator and similar devices for purposes such as the automatic dispensing of wine, the design of syphons for maintaining constant water level differences between two tanks, the opening of temple doors, and so on. These devices could be called "gadgets" since they were among the earliest examples of an idea looking for an application.

In 800 through 1200 various Arab engineers such as the three brothers Musa, Al-Jazari_, and Ibn al-Sa_'a_ti_ used float regulators for water clocks and other applications. During this period the important feedback principle of "on/off" control was used, which comes up again in connection with minimum-time problems in the 1950's.

When Baghdad fell to the Mongols in 1258 all creative thought along these lines came to an end. Moreover, the invention of the mechanical clock in the 14th century made the water clock and its feedback control system obsolete. (The mechanical clock is not a feedback control system.) The float regulator does not appear again until its use in the Industrial Revolution.

Along with a concern for his place in time, early man had a concern for his place in space. It is worth mentioning that a pseudo-feedback control system was developed in China in the 12th century for navigational purposes. The *south-pointing chariot* had a statue which was turned by a gearing mechanism attached to the wheels of the chariot so that it continuously pointed south. Using the directional information provided by the statue, the charioteer could steer a straight course. We call this a "pseudo-feedback" control system since it does not technically involve feedback unless the actions of the charioteer are considered as part of the system. Thus, it is not an automatic control system.

The Industrial Revolution

The Industrial Revolution in Europe followed the introduction of *prime movers*, or self-driven machines. It was marked by the invention of advanced grain mills, furnaces, boilers, and the steam engine. These devices could not be adequately regulated by hand, and so arose a new requirement for automatic control systems. A variety of control devices was invented, including float regulators, temperature regulators, pressure regulators, and speed control devices.

J. Watt invented his steam engine in 1769, and this date marks the accepted beginning of the Industrial Revolution. However, the roots of the Industrial Revolution can be traced back to the 1600's or earlier with the development of grain mills and the furnace.

One should be aware that others, primarily T. Newcomen in 1712, built the first steam engines. However, the early steam engines were inefficient and regulated by hand, making them less suited to industrial use. It is extremely important to realize that the Industrial Revolution did not start until the invention of improved engines and automatic control systems to regulate them.

The Millwrights

The millwrights of Britain developed a variety of feedback control devices. The fantail, invented in 1745 by British blacksmith E. Lee, consisted of a small fan mounted at right angles to the main wheel of a windmill. Its function was to point the windmill continuously into the wind.

The mill-hopper was a device which regulated the flow of grain in a mill depending on the speed of rotation of the millstone. It was in use in a fairly refined form by about 1588.

To build a feedback controller, it is important to have *adequate measuring devices*. The millwrights developed several devices for sensing speed of rotation. Using these sensors, several speed regulation devices were invented, including self-regulating windmill sails. Much of the technology of the millwrights was later developed for use in the regulation of the steam engine.

Temperature Regulators

Cornelis Drebbel of Holland spent some time in England and a brief period with the Holy Roman Emperor Rudolf II in Prague, together with his contemporary J. Kepler. Around 1624 he developed an automatic temperature control system for a furnace, motivated by his belief that base metals could be turned to gold by holding them at a precise constant temperature for long periods of time. He also used this *temperature regulator* in an incubator for hatching chickens.

Temperature regulators were studied by J.J. Becher in 1680, and used again in an incubator by the Prince de Conti and R.-A.F. de Réaumur in 1754. The "sentinel register" was developed in America by W. Henry around 1771, who suggested its use in chemical furnaces, in the manufacture of steel and porcelain, and in the temperature control of a hospital. It was not until 1777, however, that a temperature regulator suitable for industrial use was developed by Bonnemain, who used it for an incubator. His device was later installed on the furnace of a hot-water heating plant.

Float Regulators

Regulation of the level of a liquid was needed in two main areas in the late 1700's: in the boiler of a steam engine and in domestic water distribution systems. Therefore, the float regulator received new interest, especially in Britain.

In his book of 1746, W. Salmon quoted prices for ball-and-cock float regulators used for maintaining the level of house water reservoirs. This regulator was used in the first patents for the flush toilet around 1775. The flush toilet was further refined by Thomas Crapper, a London plumber, who was knighted by Queen Victoria for his inventions.

The earliest known use of a float valve regulator in a steam boiler is described in a patent issued to J. Brindley in 1758. He used the regulator in a steam engine for pumping water. S.T. Wood used a float regulator for a steam engine in his brewery in 1784. In Russian Siberia, the coal miner I.I. Polzunov developed in 1765 a float regulator for a steam engine that drove fans for blast furnaces.

By 1791, when it was adopted by the firm of Boulton and Watt, the float regulator was in common use in steam engines.

Pressure Regulators

Another problem associated with the steam engine is that of steam-pressure regulation in the boiler, for the steam that drives the engine should be at a constant pressure. In 1681 D. Papin invented a safety valve for a pressure cooker, and in 1707 he used it as a regulating device on his steam engine. Thereafter, it was a standard feature on steam engines.

The pressure regulator was further refined in 1799 by R. Delap and also by M. Murray. In 1803 a pressure regulator was combined with a float regulator by Boulton and Watt for use in their steam engines.

Centrifugal Governors

The first steam engines provided a reciprocating output motion that was regulated using a device known as a cataract, similar to a float valve. The cataract originated in the pumping engines of the Cornwall coal mines.

J. Watt’s steam engine with a rotary output motion had reached maturity by 1783, when the first one was sold. The main incentive for its development was evidently the hope of introducing a prime mover into milling. Using the rotary output engine, the Albion steam mill began operation early in 1786.

A problem associated with the rotary steam engine is that of regulating its speed of revolution. Some of the speed regulation technology of the millwrights was developed and extended for this purpose.

In 1788 Watt completed the design of the centrifugal flyball governor for regulating the speed of the rotary steam engine. This device employed two pivoted rotating flyballs which were flung outward by centrifugal force. As the speed of rotation increased, the flyweights swung further out and up, operating a steam flow throttling valve which slowed the engine down. Thus, a constant speed was achieved automatically.

The feedback devices mentioned previously either remained obscure or played an inconspicuous role as a part of the machinery they controlled. On the other hand, the operation of the flyball governor was clearly visible even to the untrained eye, and its principle had an exotic flavor which seemed to many to embody the nature of the new industrial age. Therefore, the governor reached the consciousness of the engineering world and became a sensation throughout Europe. This was the first use of feedback control of which there was popular awareness.

It is worth noting that the Greek word for governor is κ υ ßε ρ ν α ν . In 1947, Norbert Wiener at MIT was searching for a name for his new discipline of automata theory- control and communication in man and machine. In investigating the flyball governor of Watt, he investigated also the etymology of the word κ υ ßε ρ ν α ν and came across the Greek word for steersman, κ υ ßε ρ ν τ ς . Thus, he selected the name *cybernetics* for his fledgling field.

Around 1790 in France, the brothers Périer developed a float regulator to control the speed of a steam engine, but their technique was no match for the centrifugal governor, and was soon supplanted.

The Pendule Sympathique

Having begun our history of automatic control with the water clocks of ancient Greece, we round out this portion of the story with a return to mankind's preoccupation with time.

The mechanical clock invented in the 14th century is not a closed-loop feedback control system, but a precision open-loop oscillatory device whose accuracy is ensured by protection against external disturbances. In 1793 the French-Swiss A.-L. Breguet, the foremost watchmaker of his day, invented a closed-loop feedback system to synchronize pocket watches.

The pendule sympathique of Breguet used a special case of speed regulation. It consisted of a large, accurate precision chronometer with a mount for a pocket watch. The pocket watch to be synchronized is placed into the mount slightly before 12 o'clock, at which time a pin emerges from the chronometer, inserts into the watch, and begins a process of automatically adjusting the regulating arm of the watch's balance spring. After a few placements of the watch in the pendule sympathique, the regulating arm is adjusted automatically. In a sense, this device was used to transmit the accuracy of the large chronometer to the small portable pocket watch.

The Birth of Mathematical Control Theory

The design of feedback control systems up through the Industrial Revolution was by trial-and-error together with a great deal of engineering intuition. Thus, it was more of an art than a science. In the mid 1800's mathematics was first used to analyze the stability of feedback control systems. Since mathematics is the formal language of automatic control theory, we could call the period before this time the *prehistory* of control theory.

Differential Equations

In 1840, the British Astronomer Royal at Greenwich, G.B. Airy, developed a feedback device for pointing a telescope. His device was a speed control system which turned the telescope automatically to compensate for the earth's rotation, affording the ability to study a given star for an extended time.

Unfortunately, Airy discovered that by improper design of the feedback control loop, wild oscillations were introduced into the system. He was the first to discuss the *instability* of closed-loop systems, and the first to use *differential equations* in their analysis [Airy 1840]. The theory of differential equations was by then well developed, due to the discovery of the infinitesimal calculus by I. Newton (1642-1727) and G.W. Leibniz (1646-1716), and the work of the brothers Bernoulli (late 1600's and early 1700's), J.F. Riccati (1676-1754), and others. The use of differential equations in analyzing the motion of dynamical systems was established by J.L. Lagrange (1736-1813) and W.R. Hamilton (1805-1865).

Stability Theory

The early work in the mathematical analysis of control systems was in terms of differential equations. J.C. Maxwell analyzed the stability of Watt's flyball governor [Maxwell 1868]. His technique was to linearize the differential equations of motion to find the *characteristic equation* of the system. He studied the effect of the system parameters on stability and showed that the system is stable if the roots of the characteristic equation have *negative real parts*. With the work of Maxwell we can say that the theory of control systems was firmly established.

E.J. Routh provided a *numerical technique* for determining when a characteristic equation has stable roots [Routh 1877].

The Russian I.I. Vishnegradsky [1877] analyzed the stability of regulators using differential equations independently of Maxwell. In 1893, A.B. Stodola studied the regulation of a water turbine using the techniques of Vishnegradsky. He modeled the actuator dynamics and included the delay of the actuating mechanism in his analysis. He was the first to mention the notion of the *system time constant*. Unaware of the work of Maxwell and Routh, he posed the problem of determing the stability of the characteristic equation to A. Hurwitz [1895], who solved it independently.

The work of A.M. Lyapunov was seminal in control theory. He studied the stability of nonlinear differential equations using a generalized notion of energy in 1892 [Lyapunov 1893]. Unfortunately, though his work was applied and continued in Russia, the time was not ripe in the

West for his elegant theory, and it remained unknown there until approximately 1960, when its importance was finally realized.

The British engineer O. Heaviside invented operational calculus in 1892-1898. He studied the transient behavior of systems, introducing a notion equivalent to that of the *transfer function*.

System Theory

It is within the study of *systems* that feedback control theory has its place in the organization of human knowledge. Thus, the concept of a system as a dynamical entity with definite "inputs" and "outputs" joining it to other systems and to the environment was a key prerequisite for the further development of automatic control theory. The history of system theory requires an entire study on its own, but a brief sketch follows.

During the eighteenth and nineteenth centuries, the work of A. Smith in economics [*The Wealth of Nations*, 1776], the discoveries of C.R. Darwin [*On the Origin of Species By Means of Natural Selection* 1859], and other developments in politics, sociology, and elswehere were having a great impact on the human consciousness. The study of Natural Philosophy was an outgrowth of the work of the Greek and Arab philosophers, and contributions were made by Nicholas of Cusa (1463), Leibniz, and others. The developments of the nineteenth century, flavored by the Industrial Revolution and an expanding sense of awareness in global geopolitics and in astronomy had a profound influence on this Natural Philosophy, causing it to change its personality.

By the early 1900's A.N. Whitehead [1925], with his philosophy of "organic mechanism", L. von Bertalanffy [1938], with his hierarchical principles of organization, and others had begun to speak of a "general system theory". In this context, the evolution of control theory could proceed.

Mass Communication and The Bell Telephone System

At the beginning of the 20th century there were two important occurences from the point of view of control theory: the development of the telephone and mass communications, and the World Wars.

Frequency-Domain Analysis

The mathematical analysis of control systems had heretofore been carried out using differential equations in the *time domain*. At Bell Telephone Laboratories during the 1920's and 1930's, the *frequency domain* approaches developed by P.-S. de Laplace (1749-1827), J. Fourier (1768-1830), A.L. Cauchy (1789-1857), and others were explored and used in communication systems.

A major problem with the development of a mass communication system extending over long distances is the need to periodically amplify the voice signal in long telephone lines. Unfortunately, unless care is exercised, not only the information but also the noise is amplified. Thus, the design of suitable repeater amplifiers is of prime importance.

To reduce distortion in repeater amplifiers, H.S. Black demonstrated the usefulness of *negative feedback* in 1927 [Black 1934]. The design problem was to introduce a phase shift at the correct frequencies in the system. Regeneration Theory for the design of stable amplifiers was developed by H. Nyquist [1932]. He derived his *Nyquist stability criterion* based on the polar plot of a complex function. H.W. Bode in 1938 used the magnitude and phase *frequency response plots* of a complex function [Bode 1940]. He investigated closed-loop stability using the notions of *gain and phase margin*.

The World Wars and Classical Control

As mass communications and faster modes of travel made the world smaller, there was much tension as men tested their place in a global society. The result was the World Wars, during which the development of feedback control systems became a matter of survival.

Ship Control

An important military problem during this period was the control and navigation of ships, which were becoming more advanced in their design. Among the first developments was the design of sensors for the purpose of closed-loop control. In 1910, E.A. Sperry invented the *gyroscope*, which he used in the stabilization and steering of ships, and later in aircraft control.

N. Minorsky [1922] introduced his three-term controller for the steering of ships, thereby becoming the first to use the *proportional-integral-derivative (PID)* controller. He considered nonlinear effects in the closed-loop system.

Weapons Development and Gun Pointing

A main problem during the period of the World Wars was that of the accurate pointing of guns aboard moving ship and aircraft. With the publication of "Theory of Servomechanisms" by H.L. Házen [1934], the use of mathematical control theory in such problems was initiated. In his paper, Házen coined the word *servomechanisms*, which implies a master/slave relationship in systems.

The Norden bombsight, developed during World War II, used synchro repeaters to relay information on aircraft altitude and velocity and wind disturbances to the bombsight, ensuring accurate weapons delivery.

M.I.T. Radiation Laboratory

To study the control and information processing problems associated with the newly invented radar, the Radiation Laboratory was established at the Massachusetts Institute of Technology in 1940. Much of the work in control theory during the 1940's came out of this lab.

While working on an M.I.T./Sperry Corporation joint project in 1941, A.C. Hall recognized the deleterious effects of ignoring noise in control system design. He realized that the frequency-domain technology developed at Bell Labs could be employed to confront noise effects, and used this approach to design a control system for an airborne radar. This success demonstrated conclusively the importance of frequency-domain techniques in control system design [Hall 1946].

Using design approaches based on the transfer function, the block diagram, and frequency-domain methods, there was great success in controls design at the Radiation Lab. In 1947, N.B. Nichols developed his *Nichols Chart* for the design of feedback systems. With the M.I.T. work, the theory of linear servomechanisms was firmly established. A summary of the M.I.T. Radiation Lab work is provided in *Theory of Servomechanisms* [James, Nichols, and Phillips, 1947].

Working at North American Aviation, W.R. Evans [1948] presented his *root locus* technique, which provided a direct way to determine the closed-loop pole locations in the s-plane. Subsequently, during the 1950's, much controls work was focused on the s-plane, and on obtaining desirable closed-loop step-response characterictics in terms of rise time, percent overshoot, and so on.

Stochastic Analysis

During this period also, *stochastic techniques* were introduced into control and communication theory. At M.I.T in 1942, N. Wiener [1949] analyzed information processing systems using models of stochastic processes. Working in the frequency domain, he developed a *statistically optimal filter* for stationary continuous-time signals that improved the signal-to-noise ratio in a communication system. The Russian A.N. Kolmogorov [1941] provided a theory for discrete-time stationary stochastic processes.

The Classical Period of Control Theory

By now, automatic control theory using frequency-domain techniques had come of age, establishing itself as a paradigm (in the sense of Kuhn [1962]). On the one hand, a firm mathematical theory for servomechanisms had been established, and on the other, engineering design techniques were provided. The period after the Second World War can be called the *classical period* of control theory. It was characterized by the appearance of the first textbooks [MacColl 1945; Lauer, Lesnick, and Matdon 1947; Brown and Campbell 1948; Chestnut and Mayer 1951; Truxall 1955], and by straightforward design tools that provided great intuition and guaranteed solutions to design problems. These tools were applied using hand calculations, or at most slide rules, together with graphical techniques.

The Space/Computer Age and Modern Control

With the advent of the space age, controls design in the United States turned away from the frequency-domain techniques of classical control theory and back to the differential equation techniques of the late 1800's, which were couched in the *time domain*. The reasons for this development are as follows.

Time-Domain Design For Nonlinear Systems

The paradigm of classical control theory was very suitable for controls design problems during and immediately after the World Wars. The frequency-domain approach was appropriate for *linear time-invariant* systems. It is at its best when dealing with *single-input/single-output* systems, for the graphical techniques were inconvenient to apply with multiple inputs and outputs.

Classical controls design had some successes with nonlinear systems. Using the noise-rejection properties of frequency-domain techniques, a control system can be designed that is *robust* to variations in the system parameters, and to measurement errors and external disturbances. Thus, classical techniques can be used on a linearized version of a nonlinear system, giving good results at an equilibrium point about which the system behavior is approximately linear.

Frequency-domain techniques can also be applied to systems with simple types of nonlinearities using the *describing function* approach, which relies on the Nyquist criterion. This technique was first used by the Pole J. Groszkowski in radio transmitter design before the Second World War and formalized in 1964 by J. Kudrewicz.

Unfortunately, it is not possible to design control systems for advanced nonlinear multivariable systems, such as those arising in aerospace applications, using the assumption of linearity and treating the single-input/single-output transmission pairs one at a time.

In the Soviet Union, there was a great deal of activity in nonlinear controls design. Following the lead of Lyapunov, attention was focused on time-domain techniques. In 1948, Ivachenko had investigated the principle of *relay control*, where the control signal is switched discontinuously between discrete values. Tsypkin used the phase plane for nonlinear controls design in 1955. V.M. Popov [1961] provided his *circle criterion* for nonlinear stability analysis.

Sputnik - 1957

Given the history of control theory in the Soviet Union, it is only natural that the first satellite, Sputnik, was launched there in 1957. The first conference of the newly formed International Federation of Automatic Control (IFAC) was fittingly held in Moscow in 1960.

The launch of Sputnik engendered tremendous activity in the United States in automatic controls design. On the failure of any paradigm, a return to historical and natural first principles is required. Thus, it was clear that a return was needed to the time-domain techniques of the "primitive" period of control theory, which were based on differential equations. It should be realized that the work of Lagrange and Hamilton makes it straightforward to write nonlinear equations of motion for many dynamical systems. Thus, a control theory was needed that could deal with such nonlinear differential equations.

It is quite remarkable that in almost exactly 1960, major developments occurred independently on several fronts in the theory of communication and control.

Navigation

In 1960, C.S. Draper invented his inertial navigation system, which used gyroscopes to provided accurate information on the position of a body moving in space, such as a ship, aircraft, or spacecraft. Thus, the sensors appropriate for navigation and controls design were developed.

Optimality In Natural Systems

Johann Bernoulli first mentioned the *Principle of Optimality* in connection with the Brachistochrone Problem in 1696. This problem was solved by the brothers Bernoulli and by I. Newton, and it became clear that the quest for optimality is a fundamental property of motion in natural systems. Various optimality principles were investigated, including the minimum-time principle in optics of P. de Fermat (1600's), the work of L. Euler in 1744, and Hamilton's result that a system moves in such a way as to minimize the time integral of the difference between the kinetic and potential energies.

These optimality principles are all *minimum principles*. Interestingly enough, in the early 1900's, A. Einstein showed that, relative to the 4-D space-time coordinate system, the motion of systems occurs in such as way as to *maximize* the time.

Optimal Control and Estimation Theory

Since naturally-occurring systems exhibit optimality in their motion, it makes sense to design man-made control systems in an optimal fashion. A major advantage is that this design may be accomplished in the time domain. In the context of modern controls design, it is usual to minimize the time of transit, or a quadratic generalized energy functional or *performance index*, possibly with some constraints on the allowed controls.

R. Bellman [1957] applied *dynamic programming* to the optimal control of discrete-time systems, demonstrating that the natural direction for solving optimal control problems is *backwards in time*. His procedure resulted in closed-loop, generally nonlinear, feedback schemes.

By 1958, L.S. Pontryagin had developed his *maximum principle*, which solved optimal control problems relying on the *calculus of variations* developed by L. Euler (1707-1783). He solved the minimum-time problem, deriving an on/off relay control law as the optimal control [Pontryagin, Boltyansky, Gamkrelidze, and Mishchenko 1962]. In the U.S. during the 1950's, the calculus of variations was applied to general optimal control problems at the University of Chicago and elsewhere.

In 1960 three major papers were published by R. Kalman and coworkers, working in the U.S. One of these [Kalman and Bertram 1960], publicized the vital work of Lyapunov in the time-domain control of nonlinear systems. The next [Kalman 1960a] discussed the optimal control of systems, providing the design equations for the *linear quadratic regulator (LQR)*. The third paper [Kalman 1960b] discussed optimal filtering and estimation theory, providing the design equations for the *discrete Kalman filter*. The *continuous Kalman filter* was developed by Kalman and Bucy [1961].

In the period of a year, the major limitations of classical control theory were overcome, important new theoretical tools were introduced, and a new era in control theory had begun; we call it the era of *modern control*.

The key points of Kalman's work are as follows. It is a *time-domain approach*, making it more applicable for time-varying linear systems as well as nonlinear systems. He introduced *linear algebra and matrices*, so that systems with multiple inputs and outputs could easily be treated. He employed the concept of the *internal system state*; thus, the approach is one that is concerned with the internal dynamics of a system and not only its input/output behavior.

In control theory, Kalman formalized the notion of *optimality in control theory* by minimizing a very general quadratic generalized energy function. In estimation theory, he introduced stochastic notions that applied to nonstationary *time-varying systems*, thus providing a recursive solution, the Kalman filter, for the least-squares approach first used by C.F. Gauss (1777-1855) in planetary orbit estimation. The Kalman filter is the natural extension of the Wiener filter to nonstationary stochastic systems.

Classical frequency-domain techniques provide formal tools for control systems design, yet the design phase itself remained very much an art and resulted in nonunique feedback systems. By contrast, the theory of Kalman provided *optimal solutions* that yielded control systems with *guaranteed performance*. These controls were directly found by solving *formal matrix design equations* which generally had unique solutions.

It is no accident that from this point the U.S. space program blossomed, with a Kalman filter providing navigational data for the first lunar landing.

Nonlinear Control Theory

During the 1960's in the U.S., G. Zames [1966], I.W. Sandberg [1964], K.S. Narendra [Narendra and Goldwyn 1964], C.A. Desoer [1965], and others extended the work of Popov and Lyapunov in nonlinear stability. There was an extensive application of these results in the study of nonlinear distortion in bandlimited feedback loops, nonlinear process control, aircraft controls design, and eventually in robotics.

Computers in Controls Design and Implementation

Classical design techniques could be employed by hand using graphical approaches. On the other hand, modern controls design requires the solution of complicated nonlinear matrix equations. It is fortunate that in 1960 there were major developments in another area- digital computer technology. Without computers, modern control would have had limited applications.

The Development of Digital Computers

In about 1830 C. Babbage introduced modern computer principles, including memory, program control, and branching capabilities. In 1948, J. von Neumann directed the construction of the IAS stored-program computer at Princeton. IBM built its SSEC stored-program machine. In 1950, Sperry Rand built the first commercial data processing machine, the UNIVAC I. Soon after, IBM marketed the 701 computer.

In 1960 a major advance occurred- the second generation of computers was introduced which used *solid-state technology*. By 1965, Digital Equipment Corporation was building the PDP-8, and the *minicomputer* industry began. Finally, in 1969 W. Hoff invented the *microprocessor*.

Digital Control and Filtering Theory

Digital computers are needed for two purposes in modern controls. First, they are required to *solve the matrix design equations* that yield the control law. This is accomplished off-line during the design process. Second, since the optimal control laws and filters are generally time-varying, they are needed to *implement* modern control and filtering schemes on actual systems.

With the advent of the microprocessor in 1969 a new area developed. Control systems that are implemented on digital computers must be formulated in *discrete time*. Therefore, the growth of *digital control theory* was natural at this time.

During the 1950's, the theory of *sampled data systems* was being developed at Columbia by J.R. Ragazzini, G. Franklin, and L.A. Zadeh [Ragazzini and Zadeh 1952, Ragazzini and Franklin 1958]; as well as by E.I. Jury [1960], B.C. Kuo [1963], and others. The idea of using digital computers for *industrial process control* emerged during this period [Åström and Wittenmark 1984]. Serious work started in 1956 with the collaborative project between TRW and Texaco, which resulted in a computer-controlled system being installed at the Port Arthur oil refinery in Texas in 1959.

The development of *nuclear reactors* during the 1950's was a major motivation for exploring industrial process control and instrumentation. This work has its roots in the control of chemical plants during the 1940's.

By 1970, with the work of K. Åström [1970] and others, the importance of digital controls in process applications was firmly established.

The work of C.E. Shannon in the 1950's at Bell Labs had revealed the importance of sampled data techniques in the processing of signals. The applications of *digital filtering theory* were investigated at the Analytic Sciences Corporation [Gelb 1974] and elsewhere.

The Personal Computer

With the introduction of the PC in 1983, the design of modern control systems became possible for the individual engineer. Thereafter, a plethora of software control systems design packages was developed, including ORACLS, Program CC, Control-C, PC-Matlab, $MATRIX_x$, Easy5, SIMNON, and others.

The Union of Modern and Classical Control

With the publication of the first textbooks in the 1960's, modern control theory established itself as a paradigm for automatic controls design in the U.S. Intense activity in research and implementation ensued, with the I.R.E. and the A.I.E.E. merging, largely through the efforts of P. Haggerty at Texas Instruments, to form the Institute of Electrical and Electronics Engineers (I.E.E.E) in the early 1960's.

With all its power and advantages, modern control was lacking in some aspects. The guaranteed performance obtained by solving matrix design equations means that it is often possible to design a control system that works in theory without gaining any *engineering intuition* about the problem. On the other hand, the frequency-domain techniques of classical control theory impart a great deal of intuition.

Another problem is that a modern control system with any compensator dynamics can *fail to be robust* to disturbances, unmodelled dynamics, and measurement noise. On the other hand, robustness is built in with a frequency-domain approach using notions like the gain and phase margin.

Therefore, in the 1970's, especially in Great Britain, there was a great deal of activity by H.H. Rosenbrock [1974], A.G.J. MacFarlane and I. Postlethwaite [1977], and others to extend classical frequency-domain techniques and the root locus to multivariable systems. Successes were obtained using notions like the characteristic locus, diagonal dominance, and the inverse Nyquist array.

A major proponent of classical techniques for multivariable systems was I. Horowitz, whose *quantitative feedback theory* developed in the early 1970's accomplishes robust design using the Nichols chart.

In 1981 seminal papers appeared by J. Doyle and G. Stein [1981] and M.G. Safonov, A.J. Laub, and G.L. Hartmann [1981]. Extending the seminal work of MacFarlane and Postlethwaite [1977], they showed the importance of the *singular value* plots versus frequency in robust multivariable design. Using these plots, many of the classical frequency-domain techniques can be incorporated into modern design. This work was pursued in aircraft and process control by M. Athans [1986] and others. The result is a *new control theory* that blends the best features of classical and modern techniques. A survey of this *robust modern control theory* is provided by P. Dorato [1987].

1.2 THE PHILOSOPHY OF CLASSICAL CONTROL

Having some understanding of the history of automatic control theory, we may now briefly discuss the philosophies of classical and modern control theory.

Developing as it did for feedback amplifier design, classical control theory was naturally couched in the *frequency domain and the s-plane*. Relying on transform methods, it is primarily applicable for *linear time-invariant systems*, though some extensions to nonlinear systems were made using, for instance, the describing function.

The system description needed for controls design using the methods of Nyquist and Bode is the magnitude and phase of the frequency response. This is advantageous since the frequency response can be experimentally measured. The transfer function can then be computed. For root locus design, the transfer function is needed. The block diagram is heavily used to determine transfer functions of composite systems. *An exact description of the internal system dynamics is not needed* for classical design; that is, only the input/output behavior of the system is of importance.

The design may be carried out *by hand using graphical techniques*. These methods *impart a great deal of intuition* and afford the controls designer with a range of design possibilities, so that the resulting *control systems are not unique*. The design process is an engineering art.

A real system has disturbances and measurement noise, and may not be described exactly by the mathematical model the engineer is using for design. Classical theory is natural for designing control systems that are *robust* to such disorders, yielding good closed-loop performance in spite of them. Robust design is carried out using notions like the gain and phase margin.

Simple compensators like proportional-integral-derivative (PID), lead-lag, or washout circuits are generally used in the control structure. The effects of such circuits on the Nyquist, Bode, and root locus plots are easy to understand, so that a suitable compensator structure can be selected. Once designed, the compensator can be easily tuned on line.

A fundamental concept in classical control is the ability to *describe closed-loop properties in terms of open-loop properties*, which are known or easy to measure. For instance, the Nyquist, Bode, and root locus plots are in terms of the open-loop transfer function. Again, the closed-loop disturbance rejection properties and steady-state error can be described in terms of the return difference and sensitivity.

Classical control theory is *difficult to apply in multi-input/multi-output (MIMO), or multi-loop systems*. Due to the interaction of the control loops in a multivariable system, each single-input/single-output (SISO) transfer function can have acceptable properties in terms of step response and robustness, but the coordinated control motion of the system can fail to be acceptable.

Thus, classical MIMO or multiloop design requires painstaking effort using the approach of *closing one loop at a time* by graphical techniques. A root locus, for instance, should be plotted for each gain element, taking into account the gains previously selected. This is a *trial-and-error* procedure that may require multiple iterations, and it *does not guarantee good results, or even closed-loop stability*.

The multivariable frequency-domain approaches developed by the British school during the 1970's, as well as quantitative feedback theory, overcome many of these limitations, providing an effective approach for the design of many MIMO systems.

1.3 THE PHILOSOPHY OF MODERN CONTROL

Modern controls design is fundamentally a time-domain technique. An exact *state-space model* of the system to be controlled, or plant, is required. This is a first-order *vector* differential equation of the form

$dx/dt = Ax + Bu$

$y = Cx$

where x(t) is a vector of *internal variables* or system states, u(t) is a vector of control inputs, and y(t) is a vector of measured outputs. It is possible to add noise terms to represent process and measurement noises. Note that the plant is described in the time-domain.

The power of modern control has its roots in the fact that the state-space model can as well represent a MIMO system as a SISO system. That is, u(t) and y(t) are generally vectors whose entries are the individual scalar inputs and outputs. Thus, A, B, C are *matrices* whose elements describe the system dynamical interconnections.

Modern controls techniques were first firmly established for linear systems. Extensions to nonlinear systems can be made using the Lyapunov approach, which extends easily to MIMO systems, dynamic programming, and other techniques. Open-loop optimal controls designs can be determined for nonlinear systems by solving nonlinear two-point boundary-value problems.

Exactly as in the classical case, some fundamental questions on the performance of the closed-loop system can be attacked by investigating *open-loop properties*. For instance, the open-loop properties of controllability and observability of (0 (Chapter 2) give insight on what it is possible to achieve using feedback control. The difference is that, to deal with the state-space model, *a* *good knowledge of matrices and linear algebra is required.*

To achieve suitable closed-loop properties, a feedback control of the form

$u = -Kx$

may be used. The feedback gain K is a *matrix* whose elements are the individual control gains in the system. Since all the states are used for feedback, this is called *state-variable feedback*. Note that multiple feedback gains and large systems are easily handled in this framework. Thus, if there are n state components (where n can be very large in an aerospace or power distribution system) and m scalar controls, so that u(t) is an m-vector, then K is an mxn matrix with mn entries, corresponding to mn control loops.

In the standard linear quadratic regulator (LQR), the feedback gain K is chosen to minimize a quadratic time-domain *performance index (PI)* like

$$J = \int_0^{\infty} (x^T Q x + u^T R u)\, dt$$

The minimum is sought over all state trajectories. This is an extension to MIMO systems of the sorts of PIs (ITSE, ITAE, etc.) that were used in classical control. Q and R are weighting matrices that serve as *design parameters*. Their elements can be selected to provide suitable performance.

The key to LQR design is the fact that, if the feedback gain matrix K can be successfully chosen to make J finite, then the integral (0 involving the norms of u(t) and x(t) is bounded. If Q and R are correctly chosen, well-known mathematical principles then ensure that x(t) and u(t) go to zero with time. This *guarantees closed-loop stability* as well as bounded control signals in the closed-loop system.

It can be shown (see Chapter 3), that the value of K that minimizes the PI is given by

$K = R^{-1}B^TS$

where S is an nxn matrix satisfying the *Riccati equation*

$0 = A^TS + SA - SBR^{-1}B^TS + Q.$

Within this LQ framework, several points can be made. First, as long as the system (0 is controllable and Q and R are suitably chosen, the K given by these equations *guarantees the stability of the closed-loop system*

$dx/dt = (A-BK)x + Bu.$

Second, this technique is easy to apply even for multiple-input plants, since u(t) can be a vector having many components.

Third, the LQR solution relies on the solution of the *matrix design equation* (0, and so is unsuited to hand calculations. Fortunately, many design packages are by now available on digital computers for solving the Riccati design equation for S, and hence for obtaining K. Thus, *computer-aided design* is an essential feature of modern controls.

The LQR solution is a formal one that gives a *unique answer* to the feedback control problem once the design parameter Q has been selected. In fact, *the engineering art in modern design lies in the selection of the PI weighting matrices Q and R*. A body of theory on this selection process has evolved. Once Q is properly selected, the matrix design equation is formally solved for the unique K that guarantees stability.

Observe that *K is computed in terms of the open-loop quantities* A, B, Q, so that modern and classical design have this feature of determining closed-loop properties in terms of open-loop quantities in common. However, in modern control, all the entries of K are determined at the same time by using the matrix design equations. This corresponds to *closing all the feedback control loops simultaneously*, which is in complete contrast to the one-loop-at-a-time procedure of classical controls design.

Unfortunately, formal LQR design gives very *little intuition on the nature or properties of the closed-loop system*. In recent years, this deficiency has been addressed from a variety of standpoints.

Although LQR design using state feedback guarantees closed-loop stability, all the state components are seldom available for feedback purposes in a practical design problem. Therefore, *output feedback* of the form

$u = -Ky$

is more useful. LQR design equations for output feedback are more complicated than (0, but are easily derived (see Chapter 4).

Modern output-feedback design allows one to *design controllers for complicated systems with multiple inputs and outputs* by formally solving matrix design equations on a digital computer.

Another important factor is the following. While the state feedback (0 involves feedback from all states to all inputs, offering no structure in the control system, the output feedback control law (0 can be used to design a *compensator with a desired dynamical structure*, regaining much of the intuition of classical controls design.

Feedback laws like (0 and (0 are called *static*, since the control gains are constants, or at most time-varying. An alternative to static output feedback is to use a dynamic compensator of the form

$dz/dt = Fz + Gy + Eu$

$u = Hz + Dy.$

The inputs of this compensator are the system inputs and outputs. This yields a closed-loop and is called *dynamic output feedback*. The design problem is to select the matrices F, G, E, H, D for good closed-loop performance. An important result of modern control is that closed-loop stability can be guaranteed by selecting F = A-LC for some matrix L which is computed using a Riccati design equation similar to (0. The other matrices in (0 are then easily determined. This design is based on the vital *separation principle* (Chapter 10).

A disadvantage with design using F = A-LC is that then the dynamic compensator has the same number of internal states as the plant. In complicated modern aerospace and power plant applications, this dimension can be very large. Thus, various techniques for *controller reduction* and *reduced-order design* have been developed.

In standard modern control, the system is assumed to be exactly described by the mathematical model (0. In actuality, however, this model may be only an approximate description of the real plant. Moreover, in practice there can be disturbances acting on the plant, as well as measurement noise in determining y(t).

The LQR using full state feedback has some important robustness properties to such disorders, such as an infinite gain margin, 60° of phase margin, and robustness to some nonlinearities in the control loops (Chapter 10). On the other hand, the LQR using static or dynamic output feedback design has no guaranteed robustness properties. With the work on robust modern control in the early 1980's, there is now a technique (LQG/LTR, Chapter 10) for designing robust multivariable control systems. LQG/LTR design incorporates rigorous treatments of the effects of modelling uncertainties on closed-loop stability, and of disturbance effects on closed-loop performance.

With the work on robust modern design, *much of the intuition of classical controls techniques can now be incorporated in modern multivariable design*.

With modern developments in *digital control theory* and *discrete-time systems*, modern control is very suitable for the design of control systems that can be implemented on microprocessors (Part III of the book). This allows the implementation of controller dynamics that are more complicated as well as more effective than the simple PID and lead-lag structures of classical controls.

With recent work in *matrix-fraction descriptions* and *polynomial equation design*, a MIMO plant can be described not in state-space form, but in input/output form. This is a direct extension of the classical transfer function description and, for some applications, is more suitable than the internal description (0.

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