Performance metrics for Multi-Input Multi-Output (MIMO) Visible Light Communications

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Abstract—This paper reports a study of different metrics to measure the performance of different MIMO Optical Wireless geometries. Signal to Interference Noise Ratio (SINR) is found to be the most appropriate metric of those studied. The reported simulation results also illustrate that overall system performance can be approximated to that of the worst MIMO sub-channel.

Keywords-MIMO; OWC; metric; performance; geometry

I. INTRODUCTION

Visible-light optical wireless communications (VLC) has been gaining in significance, with multiple application scenarios currently being studied [1]. MIMO systems are particularly attractive for VLC as multiple LED sources are typically used and each has limited bandwidth, although this work is equally applicable to all Optical Wireless Communications (OWC) systems. Optical MIMO techniques allow design of the channel matrix, by changing the geometry of sources and detectors, but there is little work on the general relationship between overall system performance and the system geometry (studies of a specific configuration are presented in [2]). This paper presents several geometry-based metrics for measuring the performance of MIMO geometries.

II. CHANNEL MODEL AND MIMO RECOVERY

Fig.1 shows the system under consideration. An array of sources communicates with an array of detectors using an imaging MIMO arrangement, using line-of-sight (LoS) channels only. The impulse responses of the MN sub-channels can be modeled as time-invariant delta-functions [3]. The noisy signal-transmission equation for an MIMO-OWC system can be written in vector form as [4]:

$$y(t) = H \ast x(t) + n(t)$$  \hspace{1cm} (1)

For linear recovery, a matrix $G$ is needed to demultiplex received signals $y(t)$ for estimate $x'(t)$ of $x(t)$ according to:

$$x'(t) = G \ast y(t)$$  \hspace{1cm} (2)

Two common solution criteria for (2) are Zero Forcing (ZF) and Minimum Mean Squared-Error (MMSE). Mathematical forms of $G$ for those cases can be found in [5].

III. GEOMETRY-BASED METRICS

To measure and compare performance of different MIMO geometric arrangements, eligible metrics must be independent of modulation used, and based solely on the spatial configurations of the system. In (1), the channel-matrix $H$ is the only component which is a function of the system geometry (and some constant factors.) Therefore, the metrics to be found must be based on $H$, and on other signal-related factors which can be set constant for all modulation schemes.

A. Bit-Error-Rate and worst-channel domination

The Bit-Error-Rate (BER) is a modulation-dependent function of the Symbol-Error-Rate (SER), which measures performance in the actual transmission process [6]. In MIMO systems, the overall SER is the average of the individual channel SERs. The relationship between SER and channel quality represented by the Signal-to-Noise Ratio (SNR) for the $i$th channel takes the general form [6]:

$$SER_i^{chan} = \alpha \ast \text{erfc}(\Phi_i) \text{ with } \Phi_i = \beta \ast \sqrt{SNR_i^{chan}}$$  \hspace{1cm} (3)

For the same modulation scheme, SER and thus BER are functions uniquely of the SNR, as $\alpha$ and $\beta$ in (3) are dependent on modulation [6]. Therefore, geometry-only metrics to represent the system performance must correlate with SER, as the ultimate performance metric, in the same way as SNR does. Simplifying (3) with the formula in [7], it can be shown that, if the $i$th channel is such that $\Phi_k - \Phi_i \geq 4.6$ for all $k \neq i$ and for min $\Phi$ $\geq 10$, then SER$_i$ is at least 100 times larger than the SERs of all other channels. As such, the system SER is approximately dominated by SER$_i$, i.e. the worst channel.
B. Condition number

A metric based only on the matrix $\mathbf{H}$ must be able to measure how easily (2) can be performed. The condition number of $\mathbf{H}$, denoted $\text{cond}(\mathbf{H})$, is a suitable metric. (The condition number is the ratio of the largest singular value of $\mathbf{H}$ to the smallest one [8].) Larger $\text{cond}(\mathbf{H})$ leads to worse deviations of the estimate from $\mathbf{x}(\mathbf{t})$ [8].

C. Signal-to-Noise Ratio (SNR)

If ZF recovery is used, the SNR of the $i$th recovered channel takes the form [6]:

$$SNR_{i}^{\text{chan}} = \frac{\sigma_{x}^{2}}{\sigma_{n}^{2} \text{cond}(\mathbf{H})}.$$  \hspace{1cm} (4)

Equation (4) shows that for fixed signal and noise powers, SNR of a ZF-recovered channel is solely a function of the channel matrix $\mathbf{H}$ and thus is modulation-independent. It is expected that due to the similarity of (4) with that of a typical Single-Input Single-Output (SISO) channel [6], the relationship between this SNR and the SER should follow closely the prediction in (3).

D. Signal-to-Interference-and-Noise Ratio (SINR)

In MMSE recovery, since the cross-talk terms do not vanish in the estimates, the concept of Signal-to-Interference-and-Noise Ratio (SINR) is introduced to take into account the cross-talk interferences to the estimate [9]:

$$SINR_{i}^{\text{chan}} = \frac{\mathbb{E}[(\mathbf{s}_{i}^\text{desired}(\mathbf{t}))^{2}]}{\mathbb{E}[(\mathbf{s}_{i}^\text{undesired}(\mathbf{t}))^{2}]}.$$ \hspace{1cm} (6)

Mathematically closed forms of SINR for the ZF and MMSE recovery cases can be found in [9].

IV. SIMULATION AND RESULTS

A. Simulation system and methodology

Software-based simulations were performed to study the relationship between the proposed metrics and MIMO performance. Table I details the simulation configurations with arbitrary units for the powers.

Channel matrices $\mathbf{H}$ were simulated with Zemax for the optical geometries under study. Only imaging cases of the type outlined in [10] were studied; Fig.1 illustrates a typical configuration. Each of these matrices was fed into a Matlab-based system simulator to emulate the physical propagation.

The system simulator generated random data streams, multiplied them with the current channel matrix and added noises to generate the received signals. Recovery was then performed with both ZF and MMSE. From the recovered signals, BER and the proposed metrics were calculated.

B. Simulation results

For comparison, each configuration in the simulation was run with both MMSE and ZF as the MIMO-recovery algorithm. In the plots, each point represents the data of a single geometric configuration, whilst the lines (if present) represent theoretical relationships as in (3). The per-channel results are all from channel 2 (corresponding with transmitter 2) which were representative of the behaviour of all channels.

The graphs in Fig.2 show the plots of the system BER versus the condition number of the channel matrix. The imprecise nature of the BER-cond($\mathbf{H}$) correspondence (e.g. one value of cond($\mathbf{H}$) can correspond to three orders of magnitude of BER) makes cond($\mathbf{H}$) unlikely to be the desired metric.

However, Fig.3 shows well-defined relationships between cond($\mathbf{H}$) and the SINRs, depicted for the worst and best channels. The reason behind these relationships is unknown, and may be consequential of the element distribution in $\mathbf{H}$. If these SINR-cond($\mathbf{H}$) relationships are found to be more generally true, cond($\mathbf{H}$) can be a short-hand metric to the computationally-involved SINR with offsets to be determined.

The good fit between the experimental SINR-BER plot and the theoretical BER-SNR relationship for OOK in Fig.4 shows that SINR works well as the MIMO equivalent of the conventional SNR in indicating channel quality. Its modulation independence and closed-form relationship with $\text{cond}(\mathbf{H})$ suggests it as a good candidate for the design metric to optimise the channel quality.

Fig.5 depicts the plots between the system-BER and the SINR of the lowest-SINR channel, i.e. worst channel, for each geometric configuration. For both ZF and MMSE recoveries, the worst-channel-SINR vs. system-BER trends follow closely the theoretical relationship in the lower ranges of BER. This is in agreement with the prediction of worst-channel domination of performance as discussed in III.A.

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<tr>
<th>Table I. Parameters of the 6-Tx 20-Rx MIMO-OWC System</th>
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V. CONCLUSIONS AND FUTURE WORK

The simulation results show that channel cross-talk is the biggest factor limiting system performance, and the SINR is the most appropriate analytical metric for MIMO channel-performance design for the cases studied. Performance of the worst channel is found to approximate to that of the overall system. These findings form the basis of future work on designing the channel matrix by maximising the worst-channel SINR to optimise overall MIMO performance.

Figure 2. Condition number versus (a) System-BER\textsuperscript{ZF} and (b) System-BER\textsuperscript{MMSE} on log-log scales

Figure 3. Condition number (log scale) versus (a) SINR\textsuperscript{ZF} (dB) and (b) SINR\textsuperscript{MMSE} (dB)

Figure 4. Channel-BER\textsuperscript{MMSE} (log scale) versus (a) Calculated SINR\textsuperscript{MMSE} (dB) and (b) Simulated SINR\textsuperscript{MMSE} (dB)

Figure 5. System-BER (log scale) versus Worst-channel SINR (dB) for (a) ZF and (b) MMSE

REFERENCES